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Electronic transport in a randomly amplifying and absorbing chain

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Abstract : We study localization properties of a one-dimensional disordered system characterized by a random non-hermitian hamiltonian where both the randomness and the non-hermiticity arise in the local site-potential, its real part being ordered (fixed) and a random imaginary part implying the presence of either a random absorption or amplification at each site. The transmittance (forward scattering) decays exponentially in either case In contrast to the disorder in the real part of the potential (Anderson localization), the transmittance with the disordered imaginary part may decay slower than that in the case of ordered imaginary part

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The study of the spectra of systems with non-hermitian hamiltonians and of the interference of waves multiply scattered from such a system of scatterers have of late become very fashionable. The physical reason for such a description lies in the fact that the scattering in any real medium is never perfectly elastic and that in many cases the deviation from perfectly elastic scattering may be described, for example, by absorption through other inelastic channels or by amplification due to enhancement of the wave-amplitude (*e.g.*, population inversion in an active medium) of incident particles or waves. We are interested in the class of non-hermitian hamiltonians in which the non-hermiticity is in the local part (typically in one-body potentials) [1–15]. It is well-known that an imaginary term in the local part of the hamiltonian behaves like a source or a sink (depending on the sign). It may be noted that this type of complex potentials, called *optical potentials*, have been extensively studied for isolated atoms in nuclear physics. For obvious reasons, a medium having scattering potentials with positive imaginary

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part η (sink) at each site is called an *absorbing medium* and a medium with negative η (source) an *amplifying medium*.

In a disordered chain with ramdom but real-valued site-potentials, almost all the states are exponentially localized and hence an incident wave ($\sim e^{ikx}$) propagating in the positive x-direction is completely backscattered due to the well-known localization effects [16]. While a purely ordered chain with fixed absorbing site-potentials (sinks for particles) leads to an exponential decay of the transmittance (forward-scattering), one naively expects that the transmittance would increase indefinitely if each of the fixed imaginary site-potentials is amplifying (source of particles). Interestingly, it was shown by the author [7] both analytically and numerically that the transmittance asymptotically (in the large length limit) decays exponentially in both the cases and that the asymptotic decay constants are identical for an absorbing and an amplifying chain with the same magnitude ($|\eta|$) for the strength of the non-hermitian term. This somewhat surprising duality between the amplifying and the absorbing (ordered) cases was confirmed later by Paasschens *et al* [8] for a classical Helmholtz equation describing propagation of radiation (light) through a medium with a complex dielectric constant. While the above duality was originally [7] obtained for a tight binding hamiltonian, recently we [9] observed the same generic behaviour for an ordered Schrödinger hamiltonian as well. Generically, the transmittance decays monotonically with length for an absorbing chain, while it increases in an oscillatory fashion for an amplifying chain upto a length determined by $|\eta|$, beyond which the transmittance decays exponentially. The study of disorder in all the works considered so far has been constrained to the real part of the potential (dielectric constant, in the classical case). In this work, we generalize over our work in [7] and consider the effects of randomness in the amplification/absorption (*imaginary disorder*) at each site.

We consider a quantum chain of N lattice points (lattice constant unity), represented by the standard single band, tight binding equation :

$$(E-\varepsilon_n)c_n = V(c_{n-1} + c_{n+1}). \tag{1}$$

To calculate transport, we consider the *open quantum system* which consists of the above chain coupled to the external world (two reservoirs at very slightly different electrochemical potentials) with two identical semi-infinite perfect leads on either side. Here E is the fermionic energy, V is the constant nearest neighbour hopping term which is the same in both the leads and the sample, ε_n is the site-energy, and c_n is the site amplitude at the n-th site. Without any loss of generality, we choose $\varepsilon_n = 0$ in the leads and $V = 1$ to set the energy scale. Inside the sample, we choose $\varepsilon_n = \varepsilon_r + i\varepsilon_i$ where both the real and the imaginary parts could be random ($i = \sqrt{-1}$). For the purpose of this work, there is no disorder in the real part and we take for simplicity $\varepsilon_r = 0$. The imaginary part ε_i has the form $[\eta - W_i/2, \eta + W_i/2]$, where the constant part η may be either positive or negative or zero and W_i is the width of the uniform random distribution

in ε_r. The complex transmission amplitude in the ordered case ($W_i = 0$) was calculated in [7] to be

$$t_A = \frac{\left(e^{ik_s} e^{-\gamma} - e^{-ik_s} e^{\gamma}\right)\left(e^{ik} - e^{-ik}\right) e^{-ik(L+2)}}{de^{-ik_s L} e^{\gamma L} - ce^{ik_s L} e^{-\gamma L}}, \tag{2}$$

where $$c = \left(e^{ik_s - ik} e^{-\gamma} - 1\right)^2, \; d = \left(e^{-ik_s - ik} e^{\gamma} - 1\right)^2, \tag{3}$$

and the decay length $1/|\gamma| = l_a$ and the wave-vector k_s are given by

$$E = 2\cos k = \left(e^{\gamma} + e^{-\gamma}\right)\cos k_s, \tag{4}$$

and $$\eta = \left(e^{\gamma} - e^{-\gamma}\right)\sin k_s. \tag{5}$$

The transmittance or the two-probe conductance $T = g_2 = |t_A|^2$ obtained from the eq. (2) is found to decay monotonically (exponentially) towards zero for a set of absorbers ($\eta > 0$). But, for a set of amplifiers ($\eta < 0$), g_2 increases first to a high value but eventually (for large L) decays as $t_A \sim e^{-|\gamma|L}$.

Disorder in the real part (ε_r) in 1D is known to give rise to an exponential decay [16]. Let us consider the case of a disorder in the imaginary term of the potential with $\eta = 0$, but $W_i \neq 0$. It may be noted that in this case (for a long enough chain), about half of the sites act as absorbers ($\eta > 0$) and about half as amplifiers ($\eta < 0$). Then as discussed above, the net contribution to the transmittance from all the sites would essentially be decaying with a superposition of various decay constants. The modes with the fastest decay rates will possibly dominate the net transmittance. When $\eta \neq 0$, the distribution of all the decay constants will be asymmetric, and the net decay constant is

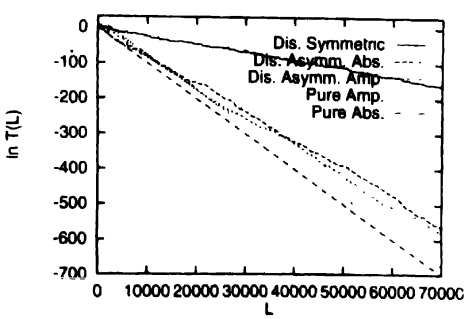

Figure 1. The variation of the logarithmic transmittance as a function of L in units of the lattice constant for various combinations of $\eta = 0$, $W_i = 0.3$. There is no disorder in the real part of the site energy. The pure absorbing/amplifying case means that $\eta = \pm 0.01$, $W_i = 0$, the symmetric disordered case means that $\eta = 0$, $W_i = 0.3$; and the asymmetric absorbing/amplifying case imply disordered cases with $\eta = \pm 0.01$, $W_i = 0.3$. Note that the transmittance decays faster in the pure cases than in the disordered ones.

expected to be somewhat different from the $\eta = 0$ case. In Figure 1, we have shown the various cases with $\eta = \pm 0.01$ and $W_i = 0.3$. We find that the pure imaginary case ($\eta = \pm 0.01$,

$W_i = 0$) gives rise to a decay length $1/2\gamma = 100$ as obtained from the equations above. For the symmetrically disordered case ($\eta = 0$, $W_i = 0.3$), the decay length is about 440. Clearly the latter decay length in the disordered case is much larger than the same for the pure imaginary case (somewhat counter-intuitive). Finally for the asymmetrically disordered case ($\eta = 0.01$, $W_i = 0.3$), the decay length is about 120 which is in between the two extreme cases.

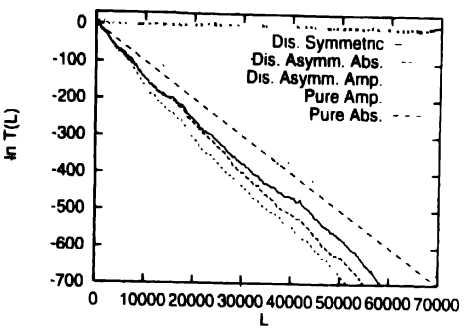

Figure 2. The same as in Figure 1, but for different combinations of $\eta = \pm 0.01$, $W_i = 0.7$. Again, there is no disorder in the real part of the site energy. For these parameters, the transmittance *decays faster for* the disordered cases than in the pure ones

In the Figure 2, we have considered another situation with the same $\eta = 0.01$ but a different disorder $W_i = 0.7$. For the symmetrical disorder case ($\eta = 0$, $W_i = 0.7$), the decay length is about 85 which is smaller than that in the pure imaginary case. This is what one normally expects to be the role played by disorder (disorder in the real part). Further, in contrast to that of Figure 1, the transmittance decays faster in the asymmetrically disordered cases ($\eta = \pm 0.01$, $W_i = 0.7$) than in the symmetrically disordered case ($\eta = 0$, $W_i = 0.7$), the decay length in the former case being about 80.

To summarise, we have studied the transmittance through a $1D$ chain with randomly amplifying and/or absorbing site-potentials at each site. We find that in contrast to the real disordered case, the decay of the transmittance (exponential localization) may be faster in the disordered case. This in particular implies that the scattering from the disorder of this type may never be incoherent. Thus there is no cut-off length scale for a crossover from localized to diffusive behaviour even when the chain consists of both amplifying and absorbing potentials.

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